How (de)coupled are Minitaur limbs?

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1 Motivation and state–of–the–art

Legged locomotion is commonly modeled as a mechanical system (namely, the body and limbs of the locomotor) subject to unilateral constraints (namely, nonpenetration constraints with respect to hard terrain) [3]. When limbs are inertially coupled, trajectory outcomes in such models vary discontinuously with respect to initial conditions [6, §3.1]; see Fig. 1(top). When limbs are inertially decoupled, trajectory outcomes vary continuously [1, Thm. 20] and (piecewise–)differentiably [5, Thm. 1] with respect to initial conditions; see Fig. 1(middle). If, in addition to being inertially decoupled, limbs are force–decoupled, then trajectory outcomes are differentiable with respect to initial conditions [4, Thm. 1]; see Fig. 1(bottom).

In light of these results, it seems advantageous to build robots with decoupled limbs, allowing the use of smooth algorithms and decreasing the sensitivity to initial conditions. Unfortunately, making limbs inertially–decoupled entails series compliance, and making limbs force–decoupled constrains controllers that sense limb/terrain contact. In either case, the robot becomes harder to control while its model becomes better behaved.

Defying the lines of reasoning in the preceding paragraphs, the Minitaur quadruped robot shown in Fig. 2a can bound, trot, and pronk, stably [2], despite seemingly obvious inertial and force coupling between limbs. This presentation addresses the question: how (de)coupled are Minitaur limbs?

2 Our approach and results

We seek to experimentally assess inertial and force (de)coupling in the Minitaur robot to determine which of several candidate models are most descriptive of the robot's pre–programmed bound, trot, and pronk behaviors. Specifically, the two candidate models for saggital plane motion we will consider are: rigid multisegment limbs with mass (Fig. 2b) and compliant (possibly massless) limbs (Fig. 2c). At first glance, the rigid limb model seems more physically accurate; since this model's limbs are inertially coupled, we would expect to experimentally observe trajectory outcomes that depend discontinuously on initial conditions. However, since the physical robots' limbs contain a small fraction ($< 10\%$) of the robot's mass, the decoupled limb model may usefully approximate the robot's motion; importantly, trajectory outcomes vary (piecewise–)differentiably with respect to initial conditions in this model.
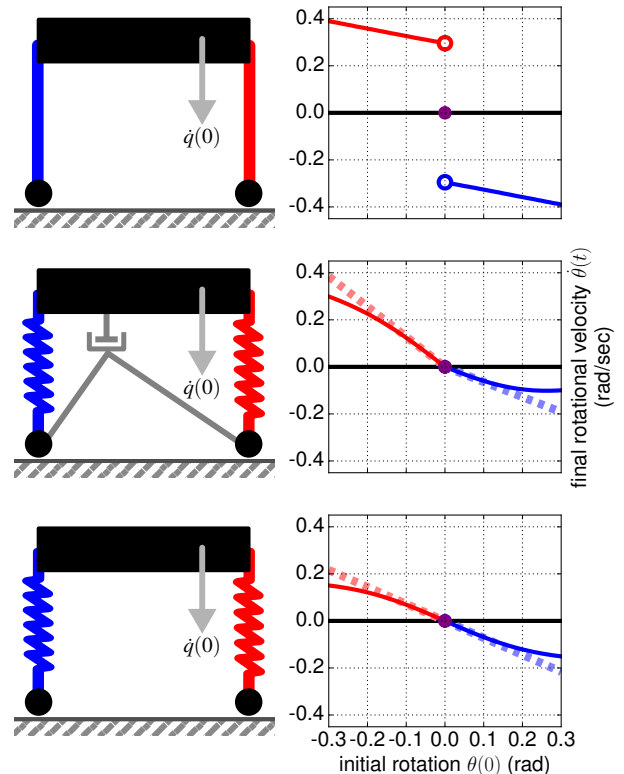

Figure 1: Trajectory outcomes in mechanical systems subject to unilateral constraints. (*top*) In general, trajectory outcomes depend discontinuously on initial conditions. In the pictured model for rigid–leg trotting (adapted from [6]), discontinuities arise when two legs touch down: if the legs impact simultaneously (corresponding to rotation $\theta(0) = 0$), then the post–impact rotational velocity is zero; if the left leg impacts before the right leg ($\theta(0) > 0$, blue) or vice–versa ($\theta(0) < 0$, red), then the post–impact rotational velocities are bounded away from zero. (*middle*) Introducing compliance into the preceding model inertially decouples the limbs from the body, thus [5, Thm. 1] implies trajectory outcomes (solid lines) are continuous and piecewise–differentiable at $\theta(0) = 0$ (dashed lines). (*bottom*) Removing coupling forces from the preceding model ensures, by virtue of [4, Thm. 1], that trajectory outcomes (solid lines) are continuous and differentiable.

We will present empirical results involving the Minitaur that test the extent to which the candidate models describe the robot's behavior. If there is a statistically significant difference between the models' descriptive power, we will quantify and report on the observed differences. If we do not observe statistically significant differences between predictions from the two models, we will conclude that the more computationally–amenable decoupled–limb model can be used with confidence in future work involving optimization and learning.

(a) photo of Minitaur quadrupedal robot
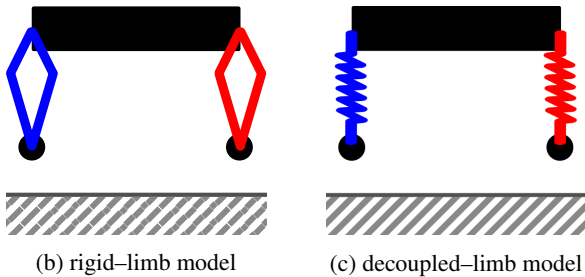
(b) rigid–limb model   (c) decoupled–limb model

Figure 2: (a) Minitaur quadrupedal robot (purchased from Ghost Robotics, LLC http://www.ghostrobotics.io/minitaur/). (b) Rigid–limb model with four limb segments and one unilateral constraint per limb. When one limb impacts the ground, an instantaneous change in velocity occurs at the other limb and the body. (c) Decoupled–limb model where each limb consists of a foot with mass connected to the body via a spring. The spring isolates the feet from the body, when one limb impacts the ground only the velocity of that limb instaneously changes. In both (b) and (c), the block box is the body and the black circles are the feet. The unilateral constraints for both models are the height of the feet.

Notes and Support

References

[1] P. Ballard. "The Dynamics of Discrete Mechanical Systems with Perfect Unilateral Constraints". In: *Archive for Rational Mechanics and Analysis* 154.3 (2000), pp. 199–274. DOI: 10.1007/s002050000105.

[2] A. De, S. A. Burden, and D. E. Koditschek. *A hybrid dynamical extension of averaging and its application to the analysis of legged gait stability. Submitted to* The International Journal of Robotics Research.

[3] A. M. Johnson, S. A. Burden, and D. E. Koditschek. "A Hybrid Systems Model for Simple Manipulation and Self–Manipulation Systems". In: *The International Journal of Robotics Research (*to appear*) (2016). URL: http://arxiv.org/abs/1502.01538.

[4] A. M. Pace and S. A. Burden. *Decoupled limbs yield differentiable trajectory outcomes through intermittent contact in locomotion and manipulation.* 2016 IEEE International Conference on Robotics and Automation (ICRA). 2017 (*to appear*). URL: https://arxiv.org/abs/1609.04056.

[5] A. M. Pace and S. A. Burden. *Piecewise–Differentiable Trajectory Outcomes in Mechanical Systems Subject to Unilateral Constraints.* Proceedings of the 20th International Conference on Hybrid Systems: Computation and Control (HSCC). 2017 (*to appear*). URL: https://arxiv.org/abs/1610.05645.

[6] C. D. Remy, K. Buffinton, and R. Siegwart. "Stability Analysis of Passive Dynamic Walking of Quadrupeds". In: *The International Journal of Robotics Research* 29.9 (2010), pp. 1173–1185. DOI: 10.1177/0278364909344635.